

Statistical reliability and path diversity based PageRank algorithm improvements

Dohy Hong

Alcatel-Lucent Bell Labs
Route de Villejust
91620 Nozay, France

dohy.hong@alcatel-lucent.com

ABSTRACT

In this paper we present new improvement ideas of the original PageRank algorithm. The first idea is to introduce an evaluation of the statistical reliability of the ranking score of each node based on the local graph property and the second one is to introduce the notion of the path diversity. The path diversity can be exploited to dynamically modify the increment value of each node in the random surfer model or to dynamically adapt the damping factor. We illustrate the impact of such modifications through examples and simple simulations.

Categories and Subject Descriptors

G.2.2 [Discrete Mathematics]: Graph Theory—*Graph algorithms*; F.2.2 [Analysis of algorithms and problem complexity]: Nonnumerical Algorithms and Problems—*Sorting and searching*; H.3.3 [Information storage and retrieval]: Information Search and Retrieval—*relevance feedback, search process*

General Terms

Algorithms

Keywords

ranking, web graph, random walk, reliability, diversity.

1. INTRODUCTION

There was an important research investment during at least 10 years on PageRank algorithm and related topics (cf. [22, 7, 14, 20, 17, 19, 4, 13, 2, 8, 23]), but there were few results concerning the original PageRank algorithm modification.

PageRank is a nice solution to evaluate the importance of the nodes of a graph based on the resolution of a fixed point problem associated to the random surfer model and to the Markov chain associated to the random walk. The PageRank algorithm can be then seen as a Perron-Frobenius problem (simplified formulation):

$$A.X = X$$

where A is the transition matrix associated to the random surfer model (the size of the state is N , if there are N URLs)

$$A(i, j) = \frac{\epsilon}{N} + (1 - \epsilon) \sum_{j=1}^N \frac{1_{j \rightarrow i}}{N(j)}$$

and X the stationary probability. X measures the relevancy of each URL (cf. [21, 15, 17]), which is proportional to the average sojourn time at each node during the random walk.

In this paper, we are interested in investigating one very specific issue which may be the Achilles' heel of PageRank. This issue is related to the possible impacts from the choice of the damping factor [12, 3, 1, 10, 9]. The role of the damping factor in the initial PageRank algorithm can be associated in the random surfer model to the probability that the surfer gets bored after several clicks and switches to a random page. More technically speaking, it may have three roles:

- [Irreducibility] firstly, it plays a role of mixing all nodes and making the associated Markov chain irreducible (i.e. we have a single connected component);
- [Indirect inheritance] secondly, it controls directly the way the importance weights are inherited when following the links (cf. illustration in Figure 1); as a consequence, it impacts the global ranking results (cf. [12, 3]);
- [Trap nodes] finally, it avoids the random walk staying too long in a trap position; the trap position can be one node (loop) or a group of nodes from which the outbound links are all local; the damping factor would enable to leave such a position and explore the whole space.

Because of the second point above, we think that the damping factor could partially induce an arbitrary ranking results, which may be undesirable. This is further illustrated in Figure 1: the node i_2 inherits most of scores from i_1 . More precisely, if the score of i_1 is $C(i_1)$, i_2 inherits from i_1 : $C(i_1)/2 \times (1 - \epsilon)$, where $1 - \epsilon$ is the damping factor and $C(i_1)$ is divided by the number of outgoing links from i_1 . Therefore, when applying PageRank family approaches this indirect influence depends directly on the value of the damping factor.

In the next sections, we present how we can correct or at least control such an impact based on the idea of the statistical reliability (Section 2) and on the idea of the path diversity (Section 3).

2. STATISTICAL RELIABILITY

We consider the random walk model on a graph \mathcal{G} of N nodes where each transition from node j to node i is defined by $p(i, j)$. In particular, we focus on a homogeneous

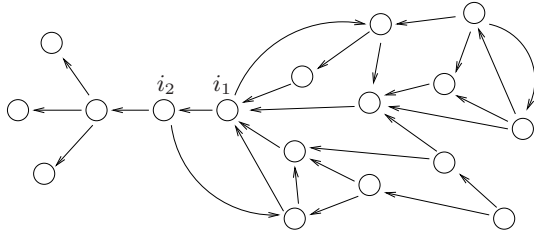


Figure 1: i_2 inherits scores from i_1

graph (one could extend the same approach on a heterogeneous graph) where $p(i, j)$ is defined by $N(j)$ the number of outgoing links from node j (if there is a link from j to i):

$$p(i, j) = \frac{\epsilon}{N} + \frac{1 - \epsilon}{N(j)}$$

where $1 - \epsilon$ is the commonly called damping factor [21].

In this section, we assume we already solved the original PageRank equation

$$X(i) = \frac{\epsilon}{N} + (1 - \epsilon) \sum_{j=1}^N \frac{1_{j \rightarrow i} X(j)}{N(j)} \quad (1)$$

to find the ranking of each node i of the graph, $X(i)$ defining the importance weight (real value between 0 and 1) of the node i .

Now, we introduce a method to evaluate the statistical reliability of $X(i)$ based on the distribution of the local incoming links's contribution.

2.1 General expression

We assume that the Markov chain associated to the random walk is described by the transition probability P with $p(i, j) = P_{i,j}$ (probability to jump from j to i) and its stationary probability $X = (x_1, \dots, x_N)$.

Then we define the quantity $r(i, j)$ by:

$$r(i, j) = p(i, j) \times x_j / x_i \quad (2)$$

By definition $\sum_{j=1}^N r(i, j) = 1$ and $r(i, j)$ can be simply interpreted as the contribution of j on x_i .

We define the following quantity measuring say the statistical error on x_i :

$$E(i) = \sum_{j=1}^N (r(i, j))^\alpha \times \beta \quad (3)$$

with $\alpha > 1$ and $\beta \in [0, 1]$ (for instance, $\beta = 0.5$ or 1 and $\alpha = 2$ which seems to be the most natural choices). And we define:

$$F(i) = 1 - E(i). \quad (4)$$

The function $F(i)$ can be interpreted as what we called the statistical reliability measure of x_i : $F(i)$ is close to one when the x_i is obtained from an equal contribution of a large number of incoming links ($F(i) = 1 - \beta/n$, if equal contribution from n links), whereas when the distribution of $r(i, j)$ is concentrated on a single node j , $F(i)$ becomes close to $1 - \beta$ which is its minimum value.

REMARK 1. In the computation of F , we can include or not the transition probability resulting from the damping factor; however, it seems more natural to exclude it, since this

is an artefact introduced for the computation and is not part of human built links.

REMARK 2. The function $F(i)$ can be also interpreted as an evaluation of the robustness of the score x_i , if for instance one local incoming link should be dropped.

2.2 Random walk based counters

Here, we assume that we maintain a counter vector C of size N to count the number of visits of all nodes during the random walk. We define a counter matrix R of size $N \times N$ and we increment the counter $R(i, j)$ by one when we jump from j to i node. If we call $C(i)$ the counter associated to the node i , the ratio $r(i, j) = R(i, j)/C(i)$ gives the contribution ratio of node j on $C(i)$. Then, we can define $F(i)$ function as above.

Based on the formula (2), one may adapt the computation of $r(i, j)$ when other strategies are used to solve the PageRank equation (1).

2.3 Exploitation of the reliability function F

The function F can be exploited for several purposes:

- for the visualization issue: when we need to show the K most relevant nodes associated to a node i , we can select the K nodes based on the K highest ratio $r(i, j)$; this idea can be generalized taking into account all distant neighbour nodes by summing the products of the form $r(i, j_1) \times r(j_1, j_2) \dots \times r(j_n, j)$ to a distant node j considering all possible path to i ; in such a generalization, we could also take into account the damping factor by multiplying by $(1 - \epsilon)^n$ depending on the length of the path n ; for the nodes pointed directly or indirectly by i , we can obviously use x_i ;
- the distribution of $r(i, j)$ may be interpreted as a statistical signature of the ranking of i and can be used to qualify the node i 's ranking and even more it can be used to modify the ranking value itself (cf. Section 4).

3. PATH DIVERSITY

Here we define the notion of the path diversity to differentiate the increment value (for the random walk based counters C , see Section 2.2). The main motivation of this approach is to avoid to give too high importance to terminal or trap positions without being forced to play with the damping factor which may have other global effect (such as what we called indirect inheritance in Section 1).

We could use more or less *aggressive* definition of the path diversity. Here we give three different formulations.

3.1 Path diversity PD1

This is the mildest version: we keep a memory of the L (if the length of the path from the initial position or the last reinitialized node is less than L , we take the path length from this position for L) last visited nodes $LP = (n_1, \dots, n_L)$ (where n_1 is the last recently visited node) and define the path diversity $div(LP)$ by the equation:

$$div(LP) = \frac{\sum_{i=1}^L f(i) \times g(i)}{\sum_{i=1}^L f(i)}, \quad (5)$$

where f and g can be defined in two ways:

Power-law model:

$$f(i) = 1/i^\alpha, \quad (6)$$

$$g(i) = 1 - \sum_{j>i, n_i=n_j} \frac{1}{(j-i)^\alpha}. \quad (7)$$

Exponential model:

$$f(i) = \gamma^i, \quad (8)$$

$$g(i) = 1 - \sum_{j>i, n_i=n_j} \delta^{j-i} \quad (9)$$

where δ should be less than 0.5. The specific choice of $\delta = 0.5$ seems the most interesting candidate (if L is very large and the path is a local loop on a same node, $g(i)$ would tend to zero).

3.2 Path diversity PD2

Here, we assume that we keep memory of the full path from the last reinitialization time (due to terminal positions or application of damping factor). If the last visited nodes are $LP = (n_1, \dots, n_L)$ (where n_1 is the last recently visited node) and the current position is n_0 , we define the path redundancy of depth i as:

$$\begin{aligned} red(i) &= g(i), \text{ if LP includes a path of length } i \text{ with a} \\ &\quad \text{first node equal to } n_0 \text{ and the last node} \\ &\quad \text{equal to } n_i, \\ &= 0, \text{ otherwise} \end{aligned}$$

and the path diversity as:

$$div(LP) = \max\left(\sum_1^\infty g(i) - \sum_1^L red(i), 0\right). \quad (10)$$

Function g can be defined in different ways, in particular we can define two types of model:

Power-law model:

$$g(i) = \frac{1}{i^\alpha}. \quad (11)$$

Exponential model:

$$g(i) = \frac{1}{\gamma^i}. \quad (12)$$

The specific choice of $\gamma = 2$ seems to be an interesting natural candidate (so that the increment is equal to 1 for a *full* diversity, i.e. all nodes are different).

3.3 Path diversity PD3

This one is probably the most *aggressive* version of diversity: this is as PD2, but with the following formulation:

$$\begin{aligned} div(LP) &= 0, \text{ if there is a node } n_i \text{ equal to } n_0, \\ &= 1, \text{ otherwise.} \end{aligned}$$

However, the impact of PD3 if we continue the random walk is not clear (we may have $div(LP)$ equal to zero than equal to 1). And such a definition would be more relevant associated with ideas of Section 3.4.

REMARK 3. *The common intuition of formulas above is to define a function that decreases as the number of unique*

elements in LP is small and with a higher impact when the duplicated node position is closer to the current position (PD1) or when the size of the duplicated jump is small (PD2).

REMARK 4. *The notion of path diversity is natural in the context of the random walk. If the PageRank equation is to be solved/computed differently, an adaptation of this approach may not be feasible and/or introduce an additional computation cost.*

REMARK 5. *In a practical solution, PD1 should have a minor impact on the global ranking, whereas PD3 will penalize the most the trap positions. With the usually applied value of the damping factor (i.e. 0.85), the depth of the graph traversal before reinitialization is small, hence PD3 definition can make sense in most of situations in a large graph.*

3.4 Other application of the path diversity

Another possible way to exploit the path diversity is to take the damping factor as a function of the path diversity, for instance with PD1, PD2 or PD3.

A simple concrete solution can be: set $\epsilon = 0$ if the current position has been already visited in the past (from the last reinitialization time): in Section 5, we show some results of such a strategy we called PR+D.

4. RANKING MODIFICATION

4.1 Reliability based modification

We propose a new adaptation of PageRank replacing the initial ranking $X(i)$ as follow (PR×F):

$$X'(i) = F(i) \times X(i). \quad (13)$$

One simple motivation of such a modification is to differentiate the case when the i 's ranking is mostly inherited from a very small number of significant neighbour nodes (say Dirac type distribution) from the case when the contribution from the neighbour nodes are spread on a large number of them (more uniform distribution). From such an information, one may decide (depends of course on the context) to credit more scores (or importances) on the nodes that depends more uniformly on a large number of nodes, which could be also a sign of the consensus on the ranking.

With such a modification, it makes sense to use a smaller damping factor than 0.85.

We further illustrate this in two simple examples below.

4.1.1 Example case C1

We set $\alpha = 2$, $\beta = 0.5$ and $\epsilon = 0.15$. In Figure 2, if we assume a has a reference score of 4 and c a score of 3 (up to a constant multiplicative factor), b inherits from a : $4 \times 0.85 = 3.4$ which is higher than c .

Applying the reliability function F : we get for a : $4 \times (1 - \beta/4) = 3.5$, b : $3.4 \times (1 - \beta) = 1.8$ and for c : $3 \times (1 - \beta/3) = 2.5$.

4.1.2 Example case C2

We set $\alpha = 2$, $\beta = 1$ and $\epsilon = 0.15$. In Figure 3: From initial PageRank: b inherits from a (assuming a score 6 for a): $6 \times 0.85 = 5.1$. Then within b, c, d , the average sojourn time

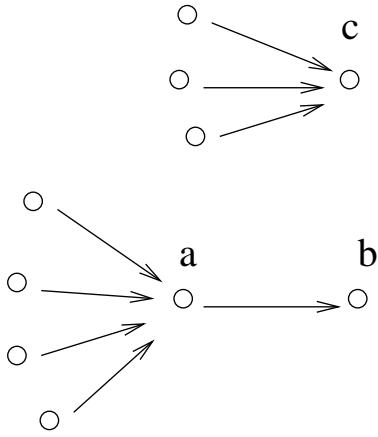


Figure 2: b inherits scores from a

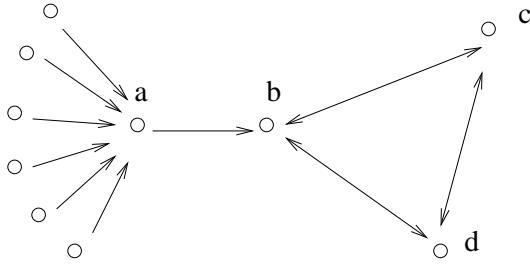


Figure 3: Case C2

before the reinitialization is $1/0.15$ to be shared between the 3 nodes. Therefore, we have for (a, b, c, d) : $(6, 7.3, 2.2, 2.2)$.

Applying the reliability function F : we get for (a, b, c, d) : $(5, 3.4, 1.1, 1.1)$.

Now with $\epsilon = 0.05$: with the initial PageRank we obtain: $(6, 12.4, 6.7, 6.7)$ and applying the reliability function we have: $(5, 8.0, 3.3, 3.3)$. This scenario shows also the necessity of keeping the damping factor not too small to avoid the deadlock position in b, c, d which tends to overestimate their importance.

The modification proposed in the next section should allow one to take ϵ close to zero without putting a too big importance weight on the nodes b, c, d .

4.2 Path diversity based modification

Here we illustrate the impact of the introduction of the path diversity in the original PageRank equation.

4.2.1 Example case C1

In this simple case, all visited nodes (before a reinitialization is required when reaching nodes b or c) are different. Therefore, the path diversity $div(LP)$ should be constant and does not impact the ranking.

4.2.2 Example case C2

In this scenario, when ϵ is close to 0, b, c, d become a trap position and their importance weights should asymptotically sum up to 1. With PD1, the importance weight of a tends to zero as well (not aggressive enough). Introducing the path diversity with PR2 or PD3, even if ϵ is equal to 0,

the increment values for b, c, d quickly tend to zero, guaranteeing a strictly positive weight of a (and of other nodes). With P+D, the importance weights of b, c, d are the most penalized.

5. SIMULATION RESULTS

Here we set a simple simulation scenario to get a first evaluation of our proposed solution and comparison to the original PageRank approach on the web graph. We don't pretend to generate any realistic model, for more details on the web graph the readers may refer to [11, 16, 5, 6, 18].

5.1 Scenario

We set N the total number of nodes (URLs) to be simulated. Then we create L random links (directional) to connect a node i to j as follow:

- the choice of the source node is done following a uniform sampling in Scenario S1 or following a power-law: $1/k^\alpha$ in Scenario S2;
- the choice of the destination node is done following a power-law: $1/k^\alpha$.

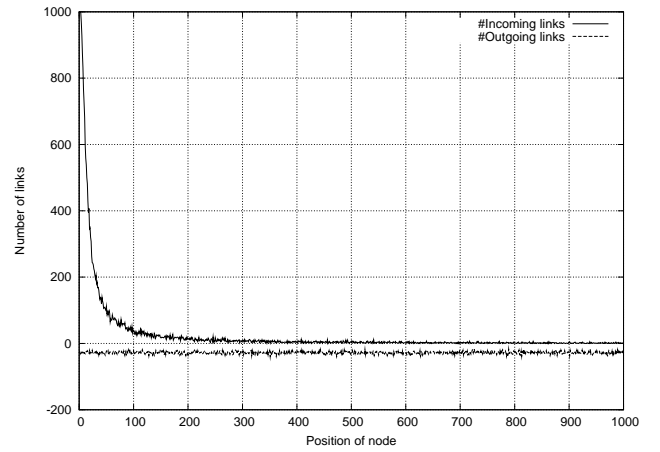


Figure 4: Scenario S1: number of incoming and outgoing links: 27880 links created.

For simplicity, we assumed no correlation between the number of incoming and outgoing links: in Scenario 1, a uniform sampling does not introduce correlation. In Scenario 2, we first associate to node k a probability proportional to $1/k^\alpha$ followed by a large number (by default N) of permutations of randomly chosen pair of nodes (i, j) : the final results define the *randomized* probability to be chosen as a source node. In both scenarios, we order the N nodes by its popularity (probability to be chosen as destination node), associating a probability proportional to $1/k^\alpha$ to the node position k : in the following, we call this the native order which is very close to the ordering by the number of incoming links (and not equal because of the random realization).

When the link already exists between the source and destination nodes, we don't modified anything (that's why we have less than L links created). The consequent results on the number of incoming and outgoing links are shown in Figures 4 and 5 ($N = 1000$ and $L = 100 \times N$, $\alpha = 1.5$).

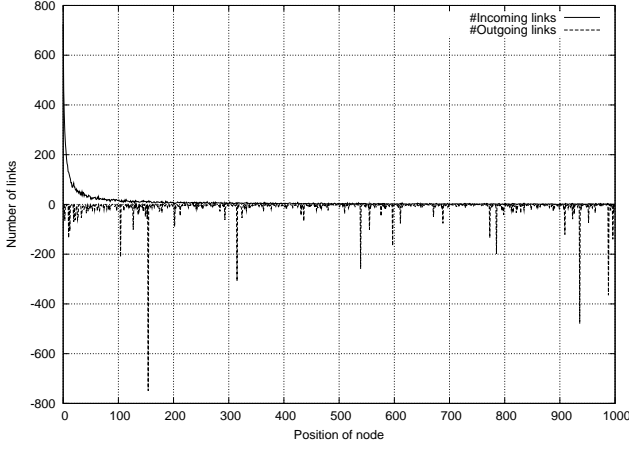


Figure 5: Scenario S2: number of incoming (same as S1) and outgoing links (N random permutations): 9533 links created.

Figure 6 shows the power-law on the number of incoming links (by construction) and on the number of nodes with k incoming links (as a consequence).

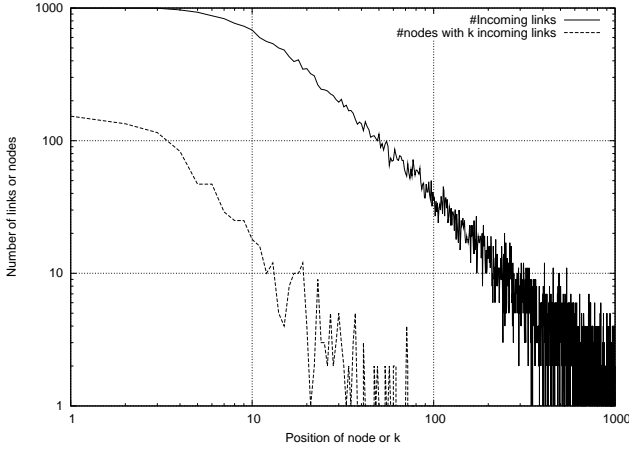


Figure 6: Scenario S1/S2: number of incoming links (w.r.t. node position) and number of nodes with k incoming links (w.r.t. k) in logscale. $\alpha = 1.5$.

5.2 Analysis

5.2.1 Scenario S1

Figure 7 shows the results of PageRank based ranking relevancy scores of the N nodes: in this case, the PageRank ranking follows closely the number of incoming links based ordering and the application of the function F merely modifies the results (we can only notice a bit more smoothed curve): because the choice of the source nodes is made randomly, the differentiation of the N nodes are only based on their difference on the probability to be chosen as destination node. So we can consider here the ranking based on the number of incoming links as the theoretically optimal one.

To evaluate the difference of the ranking scores of two ranking approaches R1 and R2 (associated to their respective normalized relevancy scores $X_1()$ and $X_2()$), we define

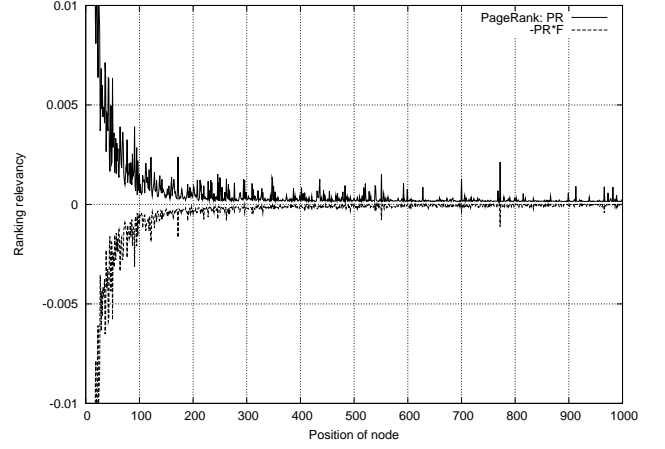


Figure 7: Scenario S1: PR and PR \times F.

the average deviation by:

$$dev(R1, R2) = \frac{1}{N} \sum_{i=1}^N \left| \sum_{k=1}^i X_1(k) - X_2(k) \right|$$

where $\sum_{k=1}^i X_1(k)$ gives the importance score of the i first nodes (following the native order).

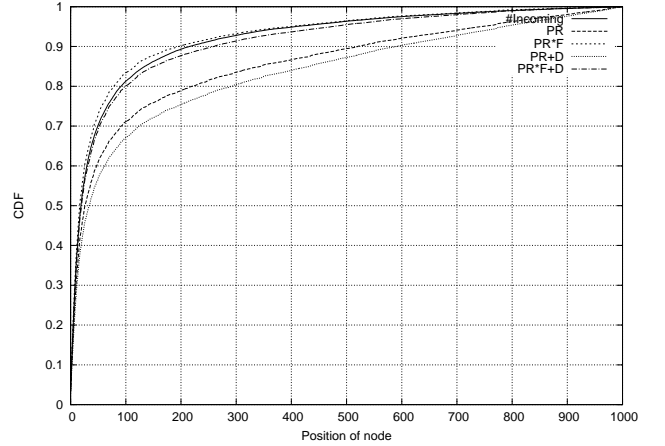


Figure 8: Scenario S1: CDF of #incoming, PR, PR \times F, PR+D and PR \times F+D.

In Table 1, we show the average deviation (w.r.t. the number of incoming links) for the four approaches. The application of F makes the scores much more closer to the ordering scores based on the number of incoming links, which is expected since the factor F will tend to favour the nodes receiving more incoming links. The application of the diversity (PR+D, cf. Section 3.4) mainly reduces here the score of the best ranked nodes and this explains its higher average deviation compared to PR.

To better highlight the difference of the ranking scores of two approaches, a node level deviation measure is defined as follows: given two ranking approaches R1 and R2, we first evaluate node per node its relevancy score ratio to the average value by: $Y(i) = X(i) \times N / \sum_i X(i)$ for R1 and R2; then we measure the deviation between R1 and R2 by $dev(i) = Y_2(i)/Y_1(i)$. The results are shown in Figure

α	PR	PR \times F	PR+D	(PR+D) \times F
1.5	0.062	0.0055	0.084	0.0077
2.0	0.071	0.0082	0.12	0.0047
2.5	0.073	0.0028	0.14	0.0017

Table 1: Average deviation.

9: we compared PR and PR \times F to the ranking score based on the number of incoming links. We observe more clearly the fact that the deviation is much more reduced when the function F is applied: this node level deviation evaluation allows one to easily observe the differences at different ranking scale. We also see that the deviation is naturally higher when there are more noises (when the number of incoming links decreases).

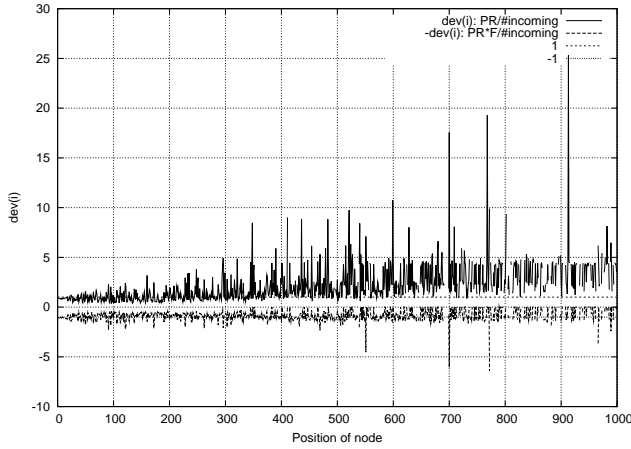


Figure 9: Scenario S1: deviation measure. (PR/#incoming) and (PR \times F/#incoming).

5.2.2 Scenario S2

Figure 10 shows the results of PageRank based ranking relevancy scores of the N nodes: in this case, there are 5 visible nodes after position 100 having a very good relevancy score with PR. Because of the relevancy inheritance of PR, even if they have few incoming links, their scores are very high when they are pointed by the best ranked nodes distributing few outgoing links. We can see that with the application of F this effect disappears.

The comparison of PR and PR \times F for the deviation measure is shown in Figure 11. We clearly see the big deviations with the 5 nodes we mentioned above.

5.2.3 Scenario S2b

The scenario S2b is as S2, except for the node 1 we imposed one outgoing link to the node 100 and the node 100 has also an unique outgoing link pointing to itself. This scenario is meant to illustrate the impact of the trap position and how we can control this impact.

Figure 12 shows the results of PR and $PR \times F$. Both results shows a very high relevancy score of the node 100. In fact, the relevancy score of the node 100 (with PR) mainly comes from the inheritance from the node 1 which can be estimated by $PR(1) \times 0.85/0.15 = 0.08785 \times 0.85/0.15 = 0.4978$ (which is very close to $x_{100} = 0.4993$). Node 100 has

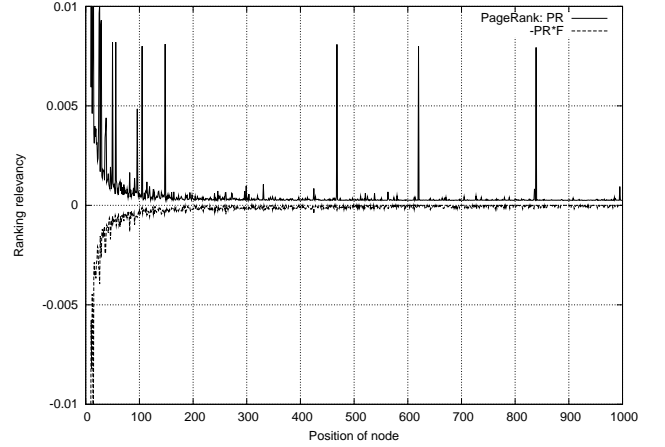


Figure 10: Scenario S2: PR and PR \times F.

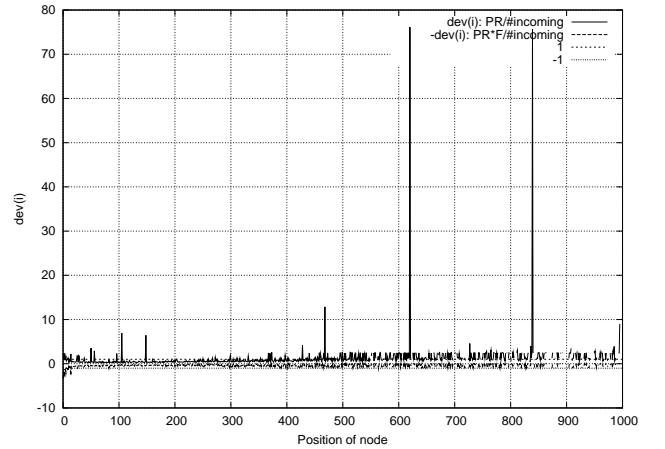


Figure 11: Scenario S2: deviation measure. (PR/#incoming) and (PR \times F/#incoming).

17 incoming links, but the main contribution is from the node 1 and as a consequence it has a small reliability score of 0.25. This reduced score decreased the importance of the node 100 (from 0.5 to 0.13), but can not control the effect of the self-pointing influence.

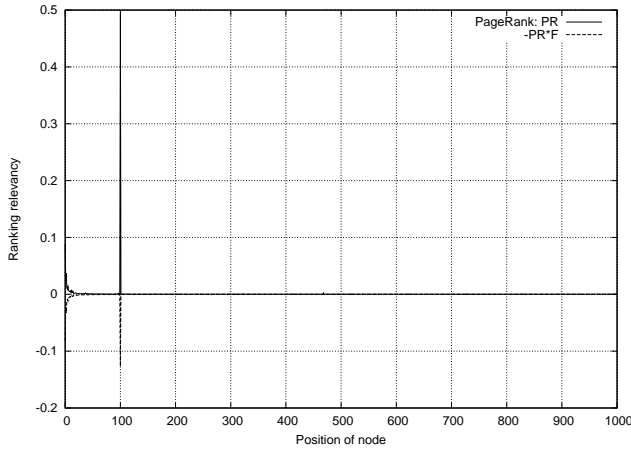


Figure 12: Scenario S2b: PR and $PR \times F$.

In Figure 13, we show the results of $PR+D$: here we see that with $PR+D$, we suppressed the self-pointing influence, but still the node 100 inherits from the first node a score of: $0.15 \times 0.85 = 0.13$ (the score of the node 1 is of course modified by modifying the damping factor value dynamically).

Now applying the function F on $PR+D$, we see that the node 100 is no more differentiated.

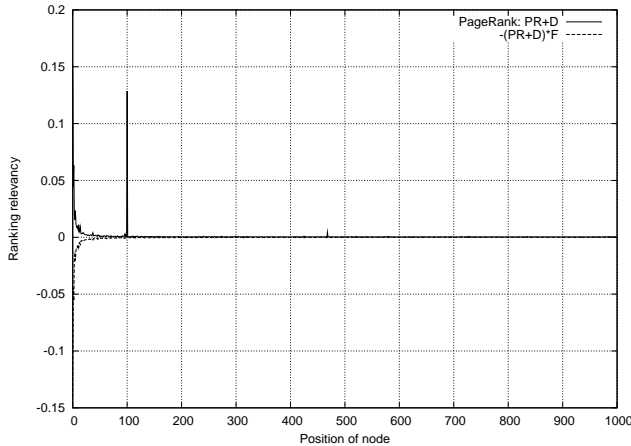


Figure 13: Scenario S2b: $PR+D$ and $(PR+D) \times F$.

Here we showed a rather extreme case, with a maximum penalty with F ($\beta = 1$) and with a maximum penalty from a non-diversity to show how much it can impact and more importantly to illustrate the fact that we can keep a control on what we called *indirect inheritance* and the impact of *trap nodes* thanks to our modifications. In a practical solution, it is necessary to correctly tune these values.

6. CONCLUSION

In this paper, we defined the statistical reliability function associated to each node of the graph and showed how it

can be applied to possibly improve the initial algorithm of PageRank results. We also discussed the benefit of introducing the notion of the path diversity to modify the increment value during the random walk or to modify the damping factor. We showed the possible consequences through simple simulation scenarios.

In a future work, we expect to test/validate those ideas through a real data based evaluation.

7. REFERENCES

- [1] K. Avrachenkov, N. Litvak, and K. S. Pham. A singular perturbation approach for choosing pagerank damping factor. *ArXiv Mathematics e-prints*, 2006.
- [2] K. Avrachenkov, B. Ribeiro, and D. Towsley. Improving random walk estimation accuracy with uniform restarts. *INRIA Technical Report RR 7394*, 2010.
- [3] R. Baeza-Yates, P. Boldi, and C. Castillo. Generalizing pagerank: damping functions for link-based ranking algorithms. *Proc. of ACM SIGIR, Conference on Research and development in information retrieval*, 2006.
- [4] A. Balmin, V. Hristidis, and Y. Papakonstantinou. Authority-based keyword queries in databases using objectrank. In *Very Large Data Bases (VLDB)*, 2004.
- [5] A.-L. Barabasi and R. Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999.
- [6] A.-L. Barabasi, R. Albert, and H. Jeong. Scale-free characteristics of random networks: the topology of the world-wide web. *Physica, A*(281):69–77, 2000.
- [7] C. T. Bergstrom. Eigenfactor: Measuring the value and prestige of scholarly journals. *C&RL News*, 68(5), 2007.
- [8] M. Bianchini, M. Gori, and F. Scarselli. Inside pagerank. *ACM Trans. Internet Techn.*, 2005.
- [9] P. Boldi, M. Santini, and S. Vigna. Pagerank as a function of the damping factor. *Proc. ACM WWW2005*, 2005.
- [10] M. Bressan and E. Peserico. Choose the damping, choose the ranking? *Proc. of WAW, LNCS 5427*:7689, 2009.
- [11] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, R. Stata, A. Tomkins, and J. Wiener. Graph structure in the web. *Computer Networks*, 33(1-6):309–320, June 2000.
- [12] H.-H. Fu, D. K. J. Lin, and H.-T. Tsai. Damping factor in google page ranking. *ppl. Stochastic Models Bus. Ind.*, 22:431444, 2006.
- [13] L. Guo, F. Shao, C. Botev, and J. Shanmugasundaram. Xrank: Ranked keyword search over xml documents. *ACM SIGMOD*, 2003.
- [14] A. Hotho, R. Jschke, C. Schmitz, and G. Stumme. Ir in folksonomies: Search and ranking. *Proc. of the 3rd Eur. Semantic Web Conf.*, 4011:411–426, 2006.
- [15] J. M. Kleinberg. Authoritative sources in a hyperlinked environment. *J. ACM*, 46(5):604–632, 1999.
- [16] R. Kumar, P. Raghavan, S. Rajagopalan, D. Sivakumar, A. Tomkins, and E. Upfal. Stochastic models for the web graph. *Proc. of the 41st Annual Symposium on Foundations of Computer Science*, 2000.

- [17] A. N. Langville and C. D. Meyer. Deeper inside pagerank. *Internet Math.*, 1(3):335–380, January 2004.
- [18] M. Levene, T. Fenner, G. Loizou, and R. Wheeldon. A stochastic model for the evolution of the web. *Computer Networks*, 39(3):277–287, June 2002.
- [19] J. Liu, W. Lai, X. sheng Hua, Y. Huang, and S. Li. Video search re-ranking via multi-graph propagation. *ACM Multimedia Conference*, pages 208–217, 2007.
- [20] Z. Nie, Y. Zhang, J.-R. Wen, and W.-Y. Ma. Object-level ranking: bringing order to web objects. *World Wide Web Conference Series*, 2005.
- [21] L. Page, S. Brin, R. Motwani, and T. Winograd. The pagerank citation ranking: Bringing order to the web. *Technical Report Stanford University*, 1998.
- [22] F. Radicchi, S. Fortunato, B. Markines, and A. Vespignani. Diffusion of scientific credits and the ranking of scientists. *Phys. Rev.*, E 80, 2009.
- [23] C.-C. Yen and J.-S. Hsu. Pagerank algorithm improvement by page relevance measurement. *Proc. FUZZ-IEEE'09, 18th international conference on Fuzzy Systems*, 2009.